A QCD Axion from Higher Dimensional Gauge Field

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(Dated: February 1, 2008)

We point out that a QCD axion solving the strong CP problem can arise naturally from parity-odd gauge field C_M in 5-dimensional (5D) orbifold field theory. The required axion coupling to the QCD anomaly comes from the 5D Chern-Simons coupling, and all other unwanted $U(1)_{PQ}$ breaking axion couplings can be avoided naturally by the 5D gauge symmetry of C_M and the 5D locality. If the fifth dimension is warped, the resulting axion scale is suppressed by small warp factor compared to the Planck scale, thereby the model can generate naturally an intermediate axion scale $f_a = 10^{10} \sim 10^{12}$ GeV.

The strong CP problem is a naturalness problem associated with that CP is conserved by the strong interactions but not by the weak interactions [1]. If the η' meson receives a mass of order Λ_{QCD} in a manner consistent with the anomalous $U(1)_A$ Ward identity, one can not avoid CP-violaton associated with the phase angle $\bar{\theta} = \theta_{QCD} + \mathrm{ArgDet}(M_u M_d)$ where θ_{QCD} is the bare QCD vacuum angle and $M_{u,d}$ denotes the 3×3 mass matrices of the up and down-type quarks. The observed CP violations in K and B-meson system suggest that $M_{u,d}$ are complex with phases of order unity, yielding the Kobayashi-Maskawa phase $\delta_{KM} \approx 1$. On the other hand, the non-observation of the neutron electric dipole moment implies $(m_u/m_d)\,|\bar{\theta}| \lesssim 10^{-10}$ for the up to down quark mass ratio m_u/m_d. This raises a question why the phase combination $\bar{\theta}$ is so small compared to the other phase combination δ_{KM}.

There are presently three possible solutions to the strong CP problem. One simple solution is that the up quark is massless, rendering all CP violations associated with $\bar{\theta}$ vanish. A massless up quark is not necessarily in conflict with the results of chiral perturbation theory since an effective up quark mass can be mimicked by instanton effects [2]. A second solution is to assume that CP is an exact symmetry of the underlying high energy theory, but is spontaneously broken in a specific manner to yield $|\bar{\theta}| \lesssim 10^{-10}$ [3]. The third, perhaps the most popular, solution is to introduce a global $U(1)_{PQ}$ symmetry which is explicitly broken by the QCD anomaly [4]. At low energies, $U(1)_{PQ}$ is non-linearly realized, leading to a light pseudo Goldstone boson [5], the axion, whose decay constant f_a is constrained as 10^{10} GeV $\lesssim f_a \lesssim 10^{12}$ GeV by astrophysical and cosmological considerations [1].

In regard to the axion solution, there are two theoretical problems to be understood. One is how to suppress the unwanted explicit breakings of $U(1)_{PQ}$ which would spoil the Peccei-Quinn mechanism for dynamical relaxation of $\bar{\theta}$ [6]. This problem becomes particularly serious when quantum gravity effects are taken into consideration. A commonly adopted idea for this problem is that there exist additional continuous and/or discrete gauge symmetries which forbid all unwanted $U(1)_{PQ}$ breakings. However if implemented within 4-dimensional (4D) effective field theory, this idea appears to be rather contrived, and the introduced gauge symmetries do not have any simple connection to the resulting axion. The second problem is how to get naturally an intermediate axion scale $f_a = 10^{10} \sim 10^{12}$ GeV, while incorporating the Planck scale $M_{Pl} \approx 10^{18}$ GeV and/or the grand unification scale $M_{GUT} \approx 10^{16}$ GeV. In this paper, we wish to point out that the enough suppression of unwanted $U(1)_{PQ}$ breakings and also an intermediate axion scale can be accomplished naturally in 5D orbifold field theories if the axion originates from a higher dimensional parity-odd gauge field C_M. As we will see, such 5D theories provide a natural framework in which the 5D gauge symmetry of C_M and the 5D locality assure that all unwanted explicit breakings of $U(1)_{PQ}$ other than the QCD anomaly are suppressed enough not to spoil the axion solution to the strong CP problem. Also if the fifth dimension is warped [7], the resulting axion decay constant is suppressed by an exponentially small warp factor compared to M_{Pl}, thereby the model can generate naturally an intermediate axion scale.

Let us briefly discuss how much should the unwanted $U(1)_{PQ}$ breakings be suppressed to keep the axion solution to work. If $U(1)_{PQ}$ is nonlinearly realized, one can always choose a field basis for which only the axion transforms under $U(1)_{PQ}$ as $a \to a +$ constant, while all other fields are invariant. Note that in this field basis, *non-derivative* axion

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couplings represent explicit $U(1)_{PQ}$ breaking. Then the axion effective action at low energy scales can be written as

$$\mathcal{L}_{\rm axion} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{1}{32\pi^2}\left(\frac{a}{f_a}+\bar{\theta}\right)\left(F\tilde{F}\right)_{QCD} + V_{\rm HE}(a/f_a)\,, \tag{1}$$

where $F\tilde{F} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$ for the gauge field strength $F^a_{\mu\nu}$ and we have ignored $U(1)_{PQ}$ invariant derivative couplings of axion. Here there are two types of $U(1)_{PQ}$ breakings: the axion coupling to the QCD anomaly $(F\tilde{F})_{QCD}$ and a high energy axion potential $V_{\rm HE}$ induced by *all other* $U(1)_{PQ}$ breaking effects such as quantum gravity effects. After integrating the QCD degrees of freedom, $(F\tilde{F})_{QCD}$ yields a low energy axion potential [1]

$$V_{\rm LE} = -f_\pi^2 m_\pi^2 \sqrt{\frac{m_u^2+m_d^2+2m_u m_d\cos(a/f_a+\bar{\theta})}{(m_u+m_d)^2}}\,, \tag{2}$$

where $f_\pi \approx 94$ MeV is the pion decay constant and $m_\pi \approx 135$ MeV is the pion mass. The conventional axion solution to the strong CP problem relies upon an assumption that $U(1)_{PQ}$ breakings other than the QCD anomaly are all suppressed enough to have

$$V_{\rm HE} \lesssim 10^{-10} f_\pi^2 m_\pi^2\,. \tag{3}$$

If this assumption holds true, the axion vacuum expectation value is determined by $V_{\rm LE}$ to cancel $\bar{\theta}$, i.e. $|\langle a\rangle/f_a + \bar{\theta}| \lesssim 10^{-10}$, thereby solves the strong CP problem. If not, the resulting QCD would not conserve CP in general. The problem is that global symmetries are broken generically by quantum gravity effects, so it is highly nontrivial to tune high energy dynamics to preserve $U(1)_{PQ}$ to an accuracy satisfying (3) [6]. Therefore, to be a true solution for the strong CP problem, one needs to provide a rationale for why all explicit breakings of $U(1)_{PQ}$ other than the QCD anomaly are so suppressed. In the following, we will see that this can be accomplished naturally in 5D orbifold field theory when the corresponding axion originates from a parity-odd 5D gauge field.

To proceed, let us consider a 5D orbifold field theory compactified on S^1/Z_2 with 5D metric $ds^2 = G_{MN}dx^M dx^N = \eta_{\mu\nu}dx^\mu dx^\nu + dy^2$ ($M,N = 0,1,2,3,5$; $\mu,\nu = 0,1,2,3$), where the fundamental domain of S^1/Z_2 is represented by $0 \le y \le \pi R$. The model contains a Z_2-odd 5D $U(1)$ gauge field C_M, Z_2-even 5D gauge fields A^a_M, generic bulk matter fields Φ and brane mater fields ϕ, obeying the following boundary conditions:

$$\begin{aligned}
&C_\mu(-y) = -C_\mu(y), \quad C_5(-y) = C_5(y), \quad C_M(y+2\pi R) = C_M(y)\,,\\
&A^a_\mu(-y) = A^a_\mu(y), \quad A^a_5(-y) = -A^a_5(y), \quad A^a_M(y+2\pi R) = A^a_M(y)\,,\\
&\Phi(-y) = \pm\Phi(y), \quad \Phi(y+2\pi R) = \pm\Phi(y).
\end{aligned}$$

Then the model possesses a $U(1)_C$ gauge symmetry under which

$$C_M \to C_M + \partial_M\Lambda\,, \quad \Phi \to e^{-iq_\Phi\epsilon(y)\Lambda}\Phi\,, \quad \phi \to \phi\,, \tag{4}$$

where q_Φ is the $U(1)_C$ charge of Φ and $\epsilon(y) = -\epsilon(-y) = \epsilon(y+2\pi R) = 1$. Here the gauge function $\Lambda(x,y)$ satisfies $\Lambda(-y) = -\Lambda(y)$ and $\Lambda(y+2\pi R) = \Lambda(y)$, so

$$\Lambda(y=0) = \Lambda(y=\pi R) = 0\,,$$

and the gauge covariant derivatives of Φ and ϕ are given by $D_M\Phi = (\nabla_M + iq_\Phi\epsilon(y)C_M)\,\Phi$ and $D_M\phi = \nabla_M\phi$ where the Z_2-even gauge fields A^a_M are contained in ∇_M. Note that all brane fields are neutral under $U(1)_C$, and Φ and $D_M\Phi$ have the same $U(1)_C$ transformation since $\Lambda(y)\frac{d}{dy}\epsilon(y) = 2\Lambda(y)\left(\delta(y)-\delta(y-\pi R)\right) = 0$. Throughout this paper, we will assume that the standard model (SM) gauge fields originate from A^a_M.

The 4D scalar field C_5 behaves like a pseudo-Goldstone boson. This feature has been used recently to get an inflaton [8], or the SM Higgs boson [9], or even a quintessence [10], from higher dimensional gauge fields. Here we wish to explore the possibility that C_5 corresponds to a QCD axion solving the strong CP problem. Basically we will examine the possible explicit breakings of $U(1)_{PQ}$ for

$$U(1)_{PQ}: \quad C_5 \to C_5 + \text{constant}. \tag{5}$$

The action of our 5D theory can be written as

$$S_{5D} = \int dy\, d^4x\, \left[\,\mathcal{L}_B + \delta(y)\,\mathcal{L}_b + \delta(y-\pi R)\,\mathcal{L}'_b\,\right]\,, \tag{6}$$

where $\mathcal{L}_B$ and $\mathcal{L}_b$, $\mathcal{L}'_b$ denote the local bulk and brane lagrangian densities, respectively. The $U(1)_C$ gauge symmetry ensures that C_M appears in the bulk lagrangian $\mathcal{L}_B$ *only* through either $C_{MN} = \partial_M C_N - \partial_N C_M$, or $D_M\Phi = (\nabla_M + iq_\Phi\epsilon(y)C_M)\Phi$, or a term of the form

$$\Delta\mathcal{L}_B = C_M\Omega^M = C_M\partial_N\omega^{MN}\,, \tag{7}$$

where Ω^M is *divergenceless*, so can be written as $\Omega^M = \partial_N\omega^{MN}$ for a two-form valued *local* functional $\omega^{MN} = -\omega^{NM}$. Note that Ω^N is required to be gauge invariant, however ω^{MN} itself does not have to be gauge invariant. Typical examples of divergenceless Ω^N with gauge *non-invariant* ω^{MN} would be $\epsilon^{MNPQR}F^a_{NP}F^a_{QR}$ and $\epsilon^{MNPQR}R_{NP}R_{QR}$ for which $C_M\Omega^M$ correspond to the well-known Chern-Simons (CS) couplings of C_M to the Yang-Mills field strength F^a_{MN} (of A^a_M) and the Riemann curvature two form R_{MN}. As for the brane lagrangians, since all bulk and brane matter fields are $U(1)_C$ invariant at $y=0$ and πR, while C_5 transforms as $C_5(0,\pi R)\to C_5(0,\pi R)+\partial_5\Lambda(0,\pi R)$, C_5 can appear in $\mathcal{L}_b$ and $\mathcal{L}'_b$ *only* through $C_{\mu 5} = \partial_\mu C_5 - \partial_5 C_\mu$.

As can be easily recognized, $U(1)_C$ and the 5D locality severely constrain the possible nonderivative couplings of C_5, and therefore $U(1)_{PQ}$ breakings, in S_{5D}. In the field basis in which $U(1)_C$ and $U(1)_{PQ}$ are realized as (4) and (5), $U(1)_{PQ}$ is broken *only* by terms involving $D_5\Phi = (\nabla_5 + q_\Phi\epsilon(y)C_5)\Phi$ or terms of the form (7). If all bulk matter fields are neutral under $U(1)_C$, i.e. all $q_\Phi = 0$, which is stable against quantum gravity effects, $U(1)_{PQ}$ would be broken only by terms of the form (7). In such case, under a $U(1)_{PQ}$ transformation $\delta C_5 = \alpha =$ constant, we have

$$\delta_{PQ}S_{5D} = \alpha\int dy\int d^4x\,\partial_\mu\omega^{5\mu}\,, \tag{8}$$

thus $U(1)_{PQ}$ *is broken only by 4D total divergence.* Symmetry breaking by 4D total divergence $\partial_\mu\omega^{5\mu}$ is dominated absolutely by the *gauge non-invariant* piece of $\omega^{5\mu}$, particularly by Yang-Mills instantons [11], since the gauge invariant piece of $\omega^{5\mu}$ vanishes rapidly at $|x|\to\infty$. We thus consider

$$\partial_\mu\omega^{5\mu} = \kappa_h\left(F\tilde{F}\right)_{\rm hidden} + \kappa_s\left(F\tilde{F}\right)_{QCD} + \kappa_w\left(F\tilde{F}\right)_{\rm weak} + ... \tag{9}$$

where the ellipsis stands for *other* 4D total divergences, e.g. $\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu}R_{\rho\sigma}$ or $\partial_\mu J^\mu$ for a gauge invariant current J^μ, which give a either null or negligible contribution to $V_{\rm HE}$ satisfying the bound (3). To be general, here we have included the instanton number of hidden gauge fields confining at $\Lambda_h \gg \Lambda_{QCD}$, as well as the instanton numbers of the QCD and electroweak gauge fields. Obviously the above breakings of $U(1)_{PQ}$ by Yang-Mills instantons arise from the CS couplings of C_M in the 5D action:

$$\int\Delta\mathcal{L}_B = \int\kappa\, C\wedge F\wedge F.$$

To solve the strong CP problem, one needs the low energy axion potential (2) induced by $(F\tilde{F})_{QCD}$, so needs $\kappa_s\neq 0$. $(F\tilde{F})_{\rm weak}$ would induce a high energy axion potential whose size is highly model-dependent [12], however the resulting $V_{\rm HE}\ll 10^{-10}f_\pi^2m_\pi^2$, so not harmful. On the other hand, $(F\tilde{F})_{\rm hidden}$ can induce $V_{\rm HE}\gg f_\pi^2m_\pi^2$, so the $U(1)_{PQ}$ breaking by $(F\tilde{F})_{\rm hidden}$ should be avoided by choosing $\kappa_h = 0$. In fact, it is a plausible assumption that the CS couplings $\kappa_{h,s,w}$ are quantized, and then the condition $\kappa_h = 0$ does not correspond to an unnatural fine tuning of parameters. Another possibility is that there are several Z_2-odd $U(1)$ gauge fields, C_M and C'_M for instance, having linearly independnet CS couplings, and then there exists a combination $\tilde{C}_5 = C_5\cos\beta + C'_5\sin\beta$ which can be identified as the QCD axion since it does not couple to $(F\tilde{F})_{\rm hidden}$, but couples to $(F\tilde{F})_{QCD}$.

So far, we have noticed that all unwanted $U(1)_{PQ}$ breakings in S_{5D} can be suppressed enough as a consequence of the 5D gauge symmetry $U(1)_C$ and the 5D locality. For this, it is required that all bulk matter fields are neutral under $U(1)_C$ and the CS coupling of C_5 to $(F\tilde{F})_{\rm hidden}$ vanishes. These conditions are stable against quantum corrections including (nonperturbative) quantum gravity effects, and thus is a natural framework. Since the fifth dimension is compact, there can be nonzero nonlocal effects breaking $U(1)_{PQ}$. However if all 5D matter fields Φ with masses $M_\Phi\lesssim\Lambda$ are neutral under $U(1)_C$, such nonlocal effects are suppressed by $e^{-\pi R\Lambda}$ where R is the orbifold radius and Λ is a cutoff scale of our 5D orbifold field theory [10]. As a result, in our case $V_{\rm HE}$ induced by nonlocal effects can be easily made to satisfy the bound (3). We thus conclude that the axion solution to the strong CP problem can be realized naturally in 5D orbifold field theory if the axion originates from a Z_2-odd 5D gauge field.

Most of low energy axion physics is determined by its decay constant f_a [1]. For a QCD axion solving the strong CP problem, cosmological and astrophysical arguments suggest $f_a = 10^{10}\sim 10^{12}$ GeV which is lower than M_{Pl} by

many orders of magnitudes. In fact, for the axion originating from 5D gauge field, a hierarchically small f_a/M_{Pl} is automatically obtained if S^1/Z_2 is *warped*. To see this, let us consider a warped geometry with $ds^2 = G_{MN}dx^M dx^N = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$ where k denotes the AdS curvature [7]. We first note that our previous arguments on the suppression of unwanted $U(1)_{PQ}$ breakings are not affected by warping. To compute f_a, we start from the 5D action

$$S_{5D} = \int d^4x\, dy \sqrt{-G}\left(\frac{1}{4g_{5C}^2}C_{MN}C^{MN} + \frac{\kappa}{\sqrt{-G}}\epsilon^{MNPQR}C_M F^a_{NP}F^a_{QR} + ...\right), \tag{10}$$

where F^a_{MN} denote the SM gauge field strengths. To get the effective 4D action of C_5, one needs to integrate out C_μ by solving its equation of motion. The equation of motion of C_μ under a y-independent background of C_5 is given by $\partial_5\left(e^{-2ky}C_{\mu 5}\right) = 0$, yielding

$$C_{\mu 5} = \partial_\mu C_5 - \partial_5 C_\mu = \frac{2\pi kRe^{2ky}}{e^{2\pi kR}-1}\,\partial_\mu C_5. \tag{11}$$

The resulting 4D effective action of C_5 is

$$S^{4D}_{\rm axion} = \int d^4x\left[\frac{\pi^2 kR^2}{2g_{5C}^2}\frac{1}{e^{2\pi kR}-1}\partial_\mu C_5\partial^\mu C_5 + 2\pi\kappa RC_5\left(F\tilde{F}\right)_{QCD}\right]. \tag{12}$$

From this, we find that the decay constant of the canonically normalized axion field is given by

$$\begin{aligned} f_a &= \frac{1}{64\pi^2\kappa}\left(\frac{k}{g_{5C}^2}\frac{1}{e^{2\pi kR}-1}\right)^{1/2} \\ &= \frac{1}{64\pi^2\kappa}\left(\frac{e^{\pi kR}}{e^{2\pi kR}-1}\right)\left(\frac{k}{g_{5C}M_5^{3/2}}\right)M_{Pl}\,, \end{aligned} \tag{13}$$

where M_5 is the 5D Planck scale and $M_{Pl}^2 = M_5^3(1-e^{-2\pi kR})/k$ [7]. Simple dimensional analysis suggests $g_{5C}^2 = \mathcal{O}(1/M_5)$ and the CS coupling $\kappa = \mathcal{O}(1)$. Then the axion decay constant of C_5 in warped geometry with $k \approx M_5$ is estimated to be

$$f_a = \mathcal{O}\left(\frac{e^{-\pi kR}M_{Pl}}{64\pi^2}\right), \tag{14}$$

so an intermediate axion scale $f_a = 10^{10} - 10^{12}$ GeV is automatically obtained when $e^{-\pi kR} = 10^{-4} - 10^{-6}$. However in this case we need $N = 1$ supersymmetry to stabilize the weak to axion scale hierarchy $M_W/f_a = 10^{-8} - 10^{-10}$. Note that if the standard model gauge fields propagate in bulk as in our case, gauge couplings run logarithmically up to energy scales of order $k \approx M_5$, so the SM gauge couplings can be unified at $M_{GUT} = \mathcal{O}(M_5) \gg f_a$ [13].

To conclude, a QCD axion solving the strong CP problem can arise naturally from a parity-odd gauge field C_M in 5D orbifold field theory. All unwanted $U(1)_{PQ}$ breakings are suppressed enough by the 5D gage symmetry of C_M and the 5D locality, while the required $U(1)_{PQ}$ breaking by the QCD anomaly originate from gauge invariant local Chern-Simons coupling of C_M. If the fifth dimension is warped, the axion decay constant of C_5 is suppressed by an exponentially small warp factor compared to M_{Pl}, while the scale of gauge unification remains to be close to M_{Pl}. Then the model can have an intermediate axion scale $f_a = 10^{10} \sim 10^{12}$ GeV together with $M_{GUT} \gg f_a$ in a natural way.

Acknowledgements

This work is supported by KRF PBRG 2002-070-C00022.

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